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The Logical Clarinet: Numerical Optimization of the Geometry of Woodwind Instruments

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Summary
The tone hole geometry of a clarinet is optimized numerically. The instrument is modeled as a network of one dimensional transmission line elements. For each (non-fork) fingering, we first calculate the resonance frequencies of the input impedance peaks, and compare them with the frequencies of a mathematically even chromatic scale (equal temperament). A least square algorithm is then used to minimize the differences and to derive the geometry of the instrument. Various situations are studied, with and without dedicated register hole and/or enlargement of the bore. With a dedicated register hole, the differences can remain less than 10 musical cents throughout the whole usual range of a clarinet. The positions, diameters and lengths of the chimneys vary regularly over the whole length of the instrument, in contrast with usual clarinets. Nevertheless, we recover one usual feature of instruments, namely that gradually larger tone holes occur when the distance to the reed increases. A fully chromatic prototype instrument has been built to check these calculations, and tested experimentally with an artificial blowing machine, providing good agreement with the numerical predictions.

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1. Introduction

Woodwind instruments of the orchestra have often attained their geometrical shapes through a slow gradual process, which in many cases has taken centuries. Guided by trial and error, skilled craftsmen have managed to develop the instruments as we know them today. In this article we study the clarinet. Most of its evolutionary process (addition of new holes and keys, etc.) was made of the succession of many small steps, each implying a limited departure from a previous configuration – for clarinets the only radical change was the introduction of the “Boehm system” of French instruments by Klosé in the middle of the 19^{th} century. A typical wind instrument has a large number of design parameters (positions and size of the holes and the chimneys, bore, etc.), while many of them contribute at the same time to the production of each note. Indeed, changing one of them in order to correct a certain note may have an unexpected, and often adverse, effect on other notes in terms of pitch, tone quality, stability, etc. In a posthumous paper, Benade [1] attempted to analyze the evolutionary path since the 18th century.

Trying new configurations by the traditional method requires a large amount of work. It therefore seems likely that the modifications tested by the instrument makers have been limited to relatively small changes, affecting only a few parameters at the same time. In other words, in terms of optimization, existing instrument designs probably represent local extrema of some optimization function, in the sense that a small change in the set of tone hole positions, radii etc. inevitably worsens the instrument. Nevertheless there might exist better geometrical shapes that are more distant in the parameter space, and therefore not accessible through small improvements of an existing design. An additional reason to believe in this scenario is given by the observation of the rather irregular tone hole pattern of many woodwinds, with alternating small and large holes, short and long chimneys, closed holes (opened for one note only) etc. It seems that no particular physical principle could explain why such an irregularity is desirable; there are actually reasons to believe that it is not, in particular if homogeneity of the production of sound over the different notes is required.

Nowadays, with mathematical models of the instrument and computer optimization algorithms, it is possible to test a number of configurations that would be inaccessible by the traditional method. It is therefore interesting to explore which results can be obtained by automatic optimization, to compare them with existing instruments, and to investigate if a strong irregularity spontaneously emerges from the optimization. The idea is not necessarily to create some completely new or exotic instrument, even if this possibil-

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ity is not excluded in the long run. It is rather to investigate whether allowing large "leaps" from usual designs leads to a completely different geometry of the instruments, to try and reach more "logical" configuration of the acoustical resonator, and eventually test them acoustically. In particular, an open question (not answered in this work) is whether or not the use of fork fingering, often used in clarinets, is an acoustical necessity, or just the result of the complicated past history of the instrument.

The purpose of this work is therefore to develop algorithms for designing, and possibly improving, woodwind instruments, in the case of the clarinet. It is to see if it is possible to conceive a "logical clarinet", with a perfectly regular fingering chart, and where the relations between the acoustical functions of the resonator and its geometry are more easy to grasp than in the traditional instrument. Of course, the instrument should produce correct pitch for all notes. Fortunately this problem is not too complicated to address in terms of calculated acoustical impedances: for simplicity it can be assumed that playing frequencies can be derived from resonance frequencies with a simple length correction in order to account for reed flow and dynamics [2]. A more difficult issue is to design an instrument with balanced timbre over its entire range. While the precise relation between tone quality and cutoff frequency of the tone hole lattice [3] is still not perfectly understood, experience seems to show that a regular cutoff frequency is useful (see [4], page 485). Here, we study the possibility of designing an instrument with a much more regular tone hole lattice in terms of tone hole diameters and positions, able to produce a complete chromatic scale over the full range of the traditional instrument.

Of course, whether such instruments will prove to be musically useful is not obvious a priori. Nevertheless, if this is the case, it is clear that interesting perspectives for making simpler and cheaper instruments could be envisaged. Our study is limited to the purely acoustical aspects of instrument design; we have not studied the problem of mechanical keys that are necessary for an instrumentalist to really play the instrument. This is indeed an important question, but this task is beyond the scope of the present work.

Numerous authors have discussed possible improvements of clarinets, in particular Benade [5], but without using numerical optimization. Brass instruments have indeed been studied by optimization [6, 7, 8], but in this case the free parameters relate to the bore of the instrument and not to the geometry of lateral holes.

This article is organized as follows. Section 2 provides the basic mathematical model used to characterize the acoustical properties of the instrument – mostly a calculation of the resonance frequencies of the resonator. Section 3 describes the optimization procedure and the minimization algorithm. Section 4 briefly discusses the computer implementation. Section 5 presents various numerical results obtained by retaining various optimization criteria; five different "clarinets" are obtained and their properties are compared. These results are used in section 6 to design an experimental prototype, and to measure its sound production with the help of an automatic blowing machine. Finally, section 7 draws a few conclusions.

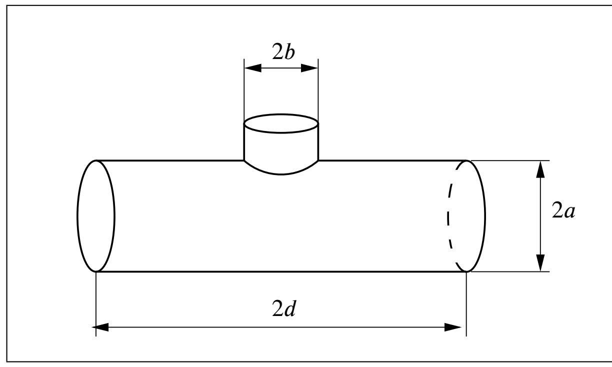

Figure 1. Elementary cell with tone hole.

2. Mathematical model

2.1. Transmission line model

The instrument is modeled with a classical one-dimensional transmission line model for planar waves [9], taking visco-thermal losses into account throughout the main bore, as well as in the tone holes. It is assumed that the distance between tone holes is sufficiently large to make higher mode interactions negligible. This assumption is valid if the distance is at least larger than the bore diameter (see e.g. [10]). Accordingly, the instrument is modeled as a succession of transfer matrices representing either a cylindrical piece of tubing, or a tone hole; each tone hole is formally represented by a lumped element.

The transfer matrix of a cylindrical piece of tubing of length L and characteristic impedance Z_c is given by

$$H = \begin{bmatrix} \cosh(\Gamma L) & Z_c \sinh(\Gamma L) \\ (1/Z_c)\sinh(\Gamma L) & \cosh(\Gamma L) \end{bmatrix}, \tag{1}$$

where Γ is the complex propagation constant. The model is rather accurate for the characteristic wavelengths propagating inside a typical wind instrument. The first higher order mode is usually far below cutoff; for a cylinder of 15 mm diameter it is a helical mode with a cutoff frequency of 13.5 kHz.

2.2. Visco-thermal boundary layer effects

The following expressions for the characteristic impedance Z_c and the wave number Γ are used (see e.g. [11])

$$Z_c = Z_0 \left[\left(1 + \frac{0.369}{r_v}\right) - \mathrm{j}\left(\frac{0.369}{r_v} + \frac{1.149}{r_v^2}\right) \right],$$
$$\Gamma = k \left(\frac{1.045}{r_v} + \frac{1.080}{r_v^2} + \mathrm{j}\left(1 + \frac{1.045}{r_v}\right) \right). \tag{2}$$

In this equation, Z_0 is equal to

$$Z_0 = \frac{\rho c}{\pi a^2}, \tag{3}$$

where ρ is the mass density of the gas, c the speed of sound and a the radius of the tube. k denotes the wavenumber ω/c, where ω is the angular frequency. The dimensionless number r_v is defined as the ratio between the tube radius and the thickness of the boundary layer

$$r_v = a\sqrt{\rho\omega/\eta}, \tag{4}$$

where η is the coefficient of viscosity.

2.3. Tone holes

Each tone hole is modeled as a T-junction (Figure 2).

The transfer matrix corresponding to this electrical equivalent circuit is the following, if $Y_s = 1/(Z_s + Z_h)$:

$$\frac{1}{1 - Y_s Z_a/4} \begin{pmatrix} 1 + Y_s Z_a/4 & Z_a \\ Y_s & 1 + Y_s Z_a/4 \end{pmatrix}. \tag{5}$$

The series impedances $Z_a/2$ are purely inertial, but the total shunt impedance Z_{st} also has a resistive part due to visco-thermal damping and radiation losses. For the acoustic masses m_a and m_s, we use expressions obtained from [12, 13],

$$\begin{aligned} m_a &= \rho t_a/(\pi a^2), \\ m_s &= \rho t_s/(\pi b^2), \end{aligned} \tag{6}$$

where

$$\begin{aligned} t_s &= b(0.82 - 0.193\delta - 1.09\delta^2 + 1.27\delta^3 - 0.71\delta^4), \\ t_a &= b(-0.37 + 0.087\delta)\delta^2, \\ \delta &= b/a. \end{aligned} \tag{7}$$

The input impedance Z_h of a tone hole of cross section area $S_h = \pi b^2$ depends on whether it is open or closed. For an open tone hole, Z_h is calculated by considering the tone hole as a transmission line terminated by a radiation impedance z_L. A simple expression for the radiation impedance of a hole in the side of a cylinder [14] is not known but, since ka is small, it seems reasonable to assume that the tone hole acts as an infinitely flanged pipe; a more detailed model for flanged termination is probably unnecessary for our purposes. At low frequencies ($ka \ll 1$), this leads to the simple formula

$$z_L = \frac{\rho c}{S_h}\left[\frac{1}{2}(ka)^2 + \mathrm{j}0.82ka\right]. \tag{8}$$

Accordingly, a tone hole of length h, terminated by an impedance z_L, is represented by the input impedance

$$Z_h = \frac{\rho c}{S_h}\frac{z_L + \mathrm{j}\frac{\rho c}{S_h}\tan(kh)}{\frac{\rho c}{S_h} + \mathrm{j}z_L\tan(kh)}. \tag{9}$$

Exterior hole interaction [15] is not taken into account; assuming that this effect remains negligible is reasonable, especially at low frequencies. The input impedance of a closed tone hole is calculated in the same way, but with $z_L \to \infty$. In the limit $kh \ll 1$, which is an acceptable approximation of the impedance for short chimneys, the closed hole input impedance expression reduces to a shunt stiffness $\rho c^2/(\mathrm{j}\omega S_h h)$.

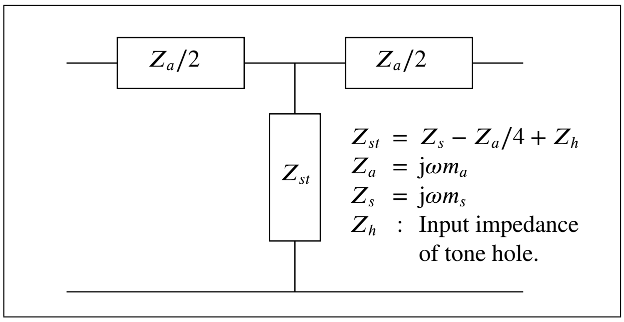

Figure 2. T-circuit equivalent for a tone hole.

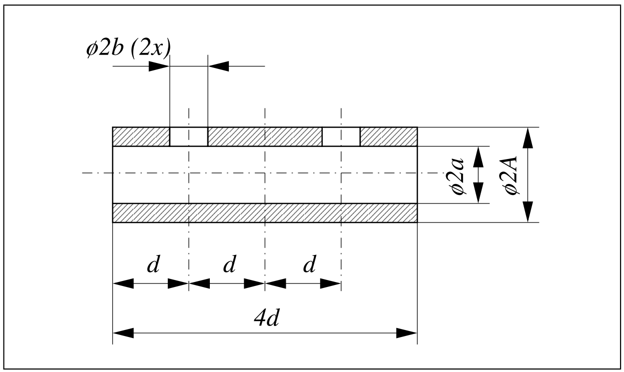

Figure 3. Instead of a bell, the main tubing is extended and fitted with two vent-holes. For $2a = 14.75$ mm, $A = 12.5$ mm, $b = 4.0$ mm, and $d = 18.2$ mm the cutoff frequency is to be $f_c = 1.420$ kHz.

2.4. Termination of the instrument

An ordinary clarinet is terminated by a bell. The main purpose of the bell is to equilibrate the timbre of the lowest notes of the instrument with that of the other notes. In this project, we replace the bell by a continuation of the cylindrical main bore with two vent-holes, as shown in Figure 3. The length of the extension and the diameters of the vent-holes are chosen in order to obtain a theoretical lattice cutoff frequency of 1.420 kHz, approximately equal to the average cutoff frequency of a clarinet [3].

2.5. Calculation of playing frequencies

The frequency of a blown note depends on the input impedance spectrum, the reed dynamics (in contact with the lips) and the blowing pressure. In practice, since the playing frequency is much smaller than the resonance frequency of the reed, the dominant factor is the input impedance.

We use a simplified description where the mouthpiece and the reed are replaced (for a given stiffness, blowing pressure, air flow correction, etc.) by an effective volume correction added to the instrument.

The effects of a temperature gradient along the air column on the pitch can be ignored in a first approximation. References [16] and [17] indicate that a satisfactory approximation of the frequency shifts is obtained by taking into account the average along the instrument of the temperature only; our calculations should then remain valid,